Multi-Commodity Maritime Inventory Routing and Scheduling

by

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Abstract

In this research, a mathematical formulation was developed for optimising the routing and scheduling of a set of heterogeneous ships carrying multiple products from various supply points to demand points. Ships have compartments for carrying multiple products between multiple supply points and multiple pick-ups/deliveries are allowed. Our formulation is an extension to those by Christiansen, Nygreen, Alkhayyal and Hwang (Christiansen, 1999; Christiansen & Nygreen, 1998a; Al-Khayyal & Hwang, 2007). A number of modifications were made to simplify the current formulation in literature by removing non-linear constraints and allow the model to be extended for more realistic multiple berths. The proposed model considers multiple commodities, multiple production points with associated production rates and various consumption points with associated consumption rates. Ships are heterogeneous and are able to carry multiple products in various compartments. Inventory levels at production and consumption points are constrained to be within specified bounds. The formulation is then solved based on a problem found in literature to obtain the optimal fleet routes and schedule, load/unload quantities and inventory levels for a planning horizon.

1. Introduction

Relative to trucking operations, maritime distribution has received much less attention. In particular, maritime inventory routing and scheduling, which is the problem of optimally routing ships, assigning departure/arrival times as well as maintaining appropriate inventory levels at harbours has not been well researched. This is despite the fact that ship fleets require larger capital and operating expenditures. Furthermore, 90% of volume and 80% of value of goods transported is carried by sea (Psaraftis, 1998). Any improvement in fleet operation may result in significant financial results which lead to a considerable need for decision support systems in ship scheduling, (Christiansen *et al*., 2004).

Very few publications have explored inventory considerations in ship routing and scheduling. These considerations are especially relevant to vertically integrated companies that operate industrial shipping operations whereby the products and ships to carry those products are owned by the same organisation. The objective of an industrial ship scheduling problem is to minimise the sum of transportation costs for all ships in the fleet (Christiansen *et al*., 2004). Typical products in these operations include pulp, slurries, petroleum and petroleum pro ducts, ore, industrial chemicals, grain and other commodities. The objective of the current work is to modify the latest formulation in literature such that it is one step closer to be applied to a particular industrial operation. Adopting the most current network flow formulation in literature, extensions were made in order to simplify the present formulation and allow it to be extended for more realistic cases in the future.

2. Literature Review

Routing problems have greatly improved trucking operations (Al-Khayyal & Hwang, 2007). For the most current review of routing problems, please see Parragh (Parragh *et al*., 2008). Shipping however has received much less attention. This is despite the fact that 90% of volume and 80% of value of goods transported is carried by sea (Psaraftis, 1998). World fleet size has experienced continuous growth (Christiansen *et al*., 2004), while seaborne trade shown similar increases. Ronen (Ronen, 1983a) included, among others, factors such as lack of problem structure, greater uncertainties and a conservative industry as reasons for a lack of work done in this area. A fleet of ships require a major capital investment and daily operating costs of a ship are up to tens of thousands of dollars (Christiansen *et al*., 2004). Improving fleet operations can lead to significant improvements in financial results (Christiansen *et al*., 2004). Therefore, there is a considerable need for, and benefits from decision support systems in ship scheduling (Christiansen *et al*., 2004). Reviews in ship routing can be found in (Ronen, 1983a,b, 1995; Christiansen *et al*., 2004; Vis & de Koster, 2003).

Long term ship routing decisions deal with strategic issues such as determination of fleet size and mix as well as permanent routing/scheduling. There are a number of works related to strategic aspects of shipping, namely Dantzig and Fulkerson, (Dantzig *et al*., 1954), Cho and Perakis (Cho & Perakis, 1996), Xinlian *et al*. (Xinlian *et al*., 2000), Fagerholt, (Fagerholt, 1999), Pesenti (Pesenti, 1995), Fagerholt and Rygh (Fagerholt & Rygh, 2002), Mehrez *et al*. (Mehrez *et al*., 1995), Darzentas and Spyrou (Darzentas & Spyrou, 1996), Imai and Rivera (Imai & Rivera, 2001), Gunnarsson *et al*. (Gunnarsson *et al*., 2006), and Crary *et al*. (Crary *et al*., 2002). Shorter term decisions, also known as operational decisions seek to find the exact routes for each ship as well as quantities to be shipped. In addition, temporal considerations are addressed in scheduling formulations and inventory levels are addressed in maritime inventory routing problems.

Ronen (Ronen, 1986) studied the scheduling of bulk or semi-bulk products from a central depot to several destinations. Heterogeneous ships with varying capacities were used. Ronen attempted and compared two heuristic methods and an optimisation algorithm based on a non-linear formulation. The optimisation algorithm, which is a biased random algorithm was found to provide good solutions that were close to optimal. Brown *et al*. (Brown *et al*., 1987) considered the routing and scheduling of crude oil between three continents. Each trip was a full shipload and consisted of a single loading port as well as a single unloading port. Constraints on loading/unloading durations were included. Brown *et al*. formulated the problem as an SP model, used enumeration to generate all feasible schedules and selected the best one. Optimal integer solutions to this problem with thousands of binary variables were obtained in less than a minute. Fisher and Rosenwein (Fisher & Rosenwein, 1989) extended this problem to allow multiple pickups from various harbours and deliveries to multiple destinations within time windows. Set partitioning was used to solve this problem. Reasonable run times were obtained and solutions achieved were projected to save up to US$30 million over the existing manual system. Hvattum *et al*. (Hvattum *et al*., 2009) looked into the problem of how to optimally allocate loads to tanks for a given ship route. A formulation was introduced as a starting point for this problem. Bausch *et al*. (Bausch *et al*., 1998) modeled the distribution of various liquid bulk products between plants, distribution centers and customers. Each ship consisted of various compartments that can be used to carry various products. Loads were pre-assigned to vessels and time windows for loading/unloading were specified. Bausch *et al*. enumerated all feasible schedules for all vessels and the best one was retained using the method described in (Brown *et al*., 1987). The optimised fleet schedule has an hourly time resolution over a planning period of two to three weeks.

Sherali *et al*. (Sherali *et al*., 1999) studied a similar problem to (Bausch *et al*., 1998) where the distribution system serves to ship crude oil and oil products from the Middle East to locations around the world. Sherali *et al*. distinguished between company controlled vessels and spot-chartered vessels. Furthermore, several routes may exist between any two harbours that have trade-o s between time and transportation costs. A heuristic was used to solve the problem which was formulated as a mixed integer program. The schedule obtained was found to be substantially better than the practice at the time. Fagerholt and Christiansen (Fagerholt & Christiansen, 2000a) explored the shipping of dry bulk products and allowed multiple pick up and delivery along with time windows. Set partitioning, using a method as described by Fagerholt and Christiansen (Fagerholt & Christiansen, 2000b) was used to solve the problem. The proposed approach was used to obtain optimal solutions for a real ship planning problem. Papadakis and Perakis (Papadakis & Perakis, 1989) studied distribution of a single bulk product to several destinations using a fleet of ships. Ships load at an origin, unload at a destination and returns to its original harbour. Optimal speeds were also considered. Papadakis and Perakis used a solution method that is based on separating the speed selection from ship allocation using Lagrangian relaxation. Jetlund and Karimi (Jetlund & Karimi, 2004) sought to determine the maximum profit schedule for delivering multiple liquid bulk products. Time windows constrain when the product is to be delivered from the load harbour to the unload harbour. Jetlund and Karimi used a heuristic method to obtain superior schedules as compared to schedules used by a shipping company. The preceding publications however do not include inventory considerations at the plants, distribution centers, or destinations. Industrial shipping operations however, are typical of vertically integrated companies which may own the products, distribution depots as well as the ships used to transport the products. As such, it is only natural to include inventory considerations at these facilities. Fagerholt and Linsdstad, (Fagerholt & Lindstad, 2007) described the implementation of an interactive decision system for ship scheduling and reported 'significant' improvements. Christiansen and Fagerholt (Christiansen & Fagerholt, 2002) used a partitioning approach for for a ship scheduling problem with time windows. The method resulted in improved schedule robustness at the price of higher transportation costs. Dauzère-Pérès *et al*., (Dauzère-Pérès *et al*., 2007) described a decision support system based on a memetic algorithm for a company supplying calcium carbonate slurry from a single processing plant. A reduction of 10% in oil consumption was reported. Korsvik and Fagerholt (Korsvik & Fagerholt, 2008) used a tabu search heuristic to maximize profit for a tramp shipping operation. The formulation allowed flexible cargo quantities and this was observed to improve the solutions. Karlaftis *et al*. (Karlaftis *et al*., 2009) used a hybrid genetic algorithm to determine the optimal routing for containerships in the Aegean Archipelago.

Many problems (40%) in ship routing and scheduling were solved using set partitioning (Christiansen *et al*., 2004). The advantage is that complex and nonlinear constraints can be incorporated during column generation and these columns can be generated via heuristics. Ship scheduling formulations are usually tightly constrained and therefore it may be possible to generate all feasible candidate schedules. If the number of feasible schedules is large then, heuristics can be used to generate promising schedules only (Christiansen *et al*., 2004).

Relatively few publications have explored inventory considerations in ship routing and scheduling. These considerations are especially relevant to vertically integrated companies that operate industrial shipping operations whereby the products and ships to carry those products are owned by the same organisation. The objective of an industrial ship scheduling problem is to minimise the sum of transportation costs for all ships in the fleet (Christiansen *et al*., 2004). Typical products in these operations include pulp, slurries, petroleum and petroleum products, ore, industrial chemicals, grain, and other commodities. Miller (Miller, 1989) originally investigated the maritime inventory routing problem. The problem addressed is a scheduling and inventory resupply problem at a chemical company that transports various chemicals from one source to multiple destinations. Inventory levels must be maintained within bounds. Miller proposed an interactive solution, using manual and automatic techniques to obtain solutions. Christiansen (Christiansen, 1999) combined ship scheduling with the problem of maintaining inventory levels at plants around the world that produce or consume a single product. Quantity loaded/unloaded must be determined by the solution. The problem was solved by decomposing the problem into two subproblems and using dynamic programming to obtain the final solution. Shen *et al*. (Shen *et al*., 2008) used the GRASP heuristic to solve the problem of transporting crude oil from a single source using a fleet of heterogeneous tankers and a pipeline to minimise inventory and transportation cost. Ronen (Ronen, 2002) studied an inventory routing problem with multiple products.

The problem is expressed as a mixed integer program and solved using a commercial optimisation suite for small instances. A heuristic was also used to obtain acceptable solutions quickly. Ronen, (Ronen, 2002) separated the shipments planning from ship scheduling and considered only discrete time dimension (days). Ship voyages have a single loading and single unloading locations. Vukadinovic and Teodorovic (Vukadinovic & Teodorovic, 1997) studied the problem of transporting gravel using inland waterway. Fuzzy logic was used to acquire the dispatcher's heuristic rules into an automatic strategy. Christiansen and Nygreen (Christiansen & Nygreen, 1998a) studied the combined inventory management and ship routing problem with time windows. The problem involved a fleet of ships carrying a single product between production and consumption harbours. Each harbour has a production or consumption rate that influenced the quantities loaded or unloaded. Ships were allowed to perform multiple pickups at various production harbours and multiple deliveries at consumption harbours. Christiansen and Nygreen formulated this problem as a mixed integer linear program and solved it using the Dantzig-Wolf decomposition and branch-and-bound. Column generation was performed using the method as described in (Christiansen & Nygreen, 1998b). This problem was then reformulated as a network model by Christiansen in (Christiansen, 1999) and solved using the method described in (Christiansen & Nygreen, 1998a). Cheng and Duran (Cheng & Duran, 2004) used Markov decision process and dynamic programming to solve small scale problems for the transportation of crude oil. Persson and Gőthe-Lundgren (Persson & Gőthe-Lundgren, 2005) studied the problem of transporting bitumen products from refineries to a set of depots. Column generation was used to solve the problem with soft inventory constraints.

Al-Khayyal and Hwang extended the model in (Christiansen & Nygreen, 1998a; Christiansen, 1999) to include multiple products. Each ship has compartments that were dedicated to specific products. The problem was formulated as a mixed integer linear program using various linearisation schemes and small instances were solved using a commercial optimisation suite. Multiple ships were allowed to arrive at a harbour at the same time, simulating multiple berths. Our formulation is a modification to those by Christiansen, Nygreen, Alkhayyal and Hwang (Christiansen, 1999; Christiansen & Nygreen, 1998a; Al-Khayyal & Hwang, 2007). Similar to previous publications, we consider the routing and scheduling for distributing multiple products using a heterogeneous fleet of ships. Ships have compartments for carrying multiple products between multiple supply points and multiple pick-ups/deliveries are allowed.

In maritime routing and scheduling literature, a number of overlapping terms are commonly used that require clarification, including ports, harbours, terminals and berths. Harbour is a general area where ships can be accommodated and can be natural or man-made. A man-made harbour is known as a port. A terminal is a built facility where products are transferred from/to ships. For the purpose of this paper, harbours, ports and terminals will be considered synonymous. A berth is a specific location within a harbour or port for ships to moor for the purpose for loading or unloading products. Structures that may be considered synonymous to berths within the context of mathematical formulations presented here are quays, wharfs, jetties and piers.

3. Mathematical Formulation

The formulation presented here is developed along the lines of Christiansen, Nygreen, Al-Khayyal and Hwang (Christiansen, 1999; Christiansen & Nygreen, 1998a; Al-Khayyal & Hwang, 2007), with modifications to prevent undesirable results, simplify as well tighten various constraints, and allow the model to be extended for more realistic multiple berths. The formulation is largely adopted from, and therefore closely resembles the one in Al-Khayyal and Hwang (Al-Khayyal & Hwang, 2007), the most recent publication in this area. Explanation of the constraints is also adopted accordingly, while additions to constraints are explained in the same format.

The distribution system consists of a set of harbours, a set of products and a set of ships to carry products between harbours. The average production and consumption rate for each product at each harbour within the planning horizon is known. Ships are compartmentalised and may carry more than one product at a time. Also, ships are heterogeneous with varying capacities for different products, different cost parameters, and are able to carry different sets of products. Inventory levels at each harbour must be maintained within the storage capacity. A ship may perform multiple pick-ups and multiple drop-offs at various harbours. The time it takes to load/unload products at each harbour is known. The objective is to obtain the routing and schedule for each ship as well as quantities to be loaded/unloaded that minimises the total fuel costs, port and canal dues, and loading/unloading charges over a planning horizon.

The distribution system is modeled as an arc flow formulation. Constraints for routing, loading/unloading, time aspects and inventory are then added to the arc flow formulation. Similar to the convention used in (Al-Khayyal & Hwang, 2007), decision variables are presented in lower case letters while parameters and sets are in upper case.

3.1 Routing

The basis of the formulation is a network with nodes (i,m), where i signifies a harbour, and m signifies the arrival number at a particular harbour. The set element (i,m) is called a position. Each harbour has a specified set of available arrival numbers $\{1, 2, ..., \mu\}$. H_T is the set of all harbours, M the set of available numbers and S the set of possible harbour-arrival pairs; i.e.,

$S_T = \{(i,m) | i \in H_T, m \in M_i\}$. A_v is the set of all feasible arcs for ship v.

At the beginning at each planning horizon, each ship is assumed to be occupying an initial position $(i_v, m_v) \in S_T$. If ship 2 is located at harbour 3 for instance, then $i_2 = 3$, and if it is the only ship at that harbour at the beginning of the planning horizon, then $m_v = 1$. The set of all initial positions is then $S := \{(i,m) \mid v \in V\}$. Further, $S := S_T \setminus S_0$ is the set of new available positions that ships can occupy after leaving their initial positions. The arc flow variable x_{imjnv} is set equal to 1 if ship $v \in V$ travels from position (i,m) to position (j,n) where $(i,m) \in S_T$ and (j, n) S_N.

Initial position The route end variables z_{imv} is set to 1 if ship v ends its route at position (i,m). Constraints (1) require that the arc flow variable be set to 1 as ship v travels from initial position (i,m) to another position (j,n). If the ship does not depart to other harbours then $z_{i_v m_v v} = 1$.

$$\sum_{(j,n)\in S_N} x_{i_v m_v jnv} + z_{i_v m_v v} = 1, \text{ for every } v \in V. \tag{1}$$

Route continuation Constraints (2) ensure that if ship v arrives at position (i,m), then there is a corresponding departure from that position, x_{imjnv}, or the ship's journey may be terminated at (i,m) by setting $z_{imv} = 1$.

$$\sum_{(j,n)\in S_T} x_{jnimv} - \sum_{(j,n)\in S_N} x_{imjnv} + z_{imv} = 0, \text{ for every } (v,i,m) \in V \times S_N. \tag{2}$$

Route finishing constraints that ensure that ships terminate their journeys in harbours at the end of the planning horizon are omitted here since in practice, ships may be mid-ocean at the end of the period.

Single visit Constraints (3) require that position (i,m) is visited at most once. If (i,m) is not visited at all during the planning period, then the slack variable y_{im} is set to 1.

$$\sum_{v\in V} \sum_{(j,n)\in S_T} x_{jnimv} + y_{im} = 1, \text{ for every } (i,m) \in S_N. \tag{3}$$

Intra-harbour travel Constraints (4) prevent ships from moving between different positions in the same harbour; i.e., ships can only travel between different harbours. Without these constraints, numerical experiments resulted in ships moving within the same harbour.

$$x_{imjnv} = 0, \text{ for every } (i,m) \in S_T, (j,n) \in S_T, v \in V \text{ and } i = j. \tag{4}$$

Position sequence Constraints (5) require that positions are sequentially occupied. The first arrival to a harbour should occupy position $m = 1$, the second, position $m = 2$ and so on. Constraints (5) ensure that there will not be a case of position $m = 2$ being occupied while position $m = 1$ being empty.

$$y_{im} - y_{i(m-1)} \geq 0, \text{ for every } (i,m) \in S_N. \tag{5}$$

3.2 Loading and unloading

A number of parameters are introduced to specify the loading and unloading constraints. J_{ik} is specified as equal to +1 if harbour i is a producer of product k, and -1 if it is a consumer. q_{imvk} is the quantity of product k loaded onto, or unloaded from ship v depending on J at that harbour. l_{imvk} is the quantity of product k onboard ship v at the moment it departs position m at harbour i. Servicing variable o_{imvk} is set to 1 if product k is serviced at harbour i and 0 otherwise. Q_{vk} is the initial quantity of product k already loaded onto ship v at the start of the planning horizon. CAP_{vk} is the capacity of a dedicated compartment on ship v to carry product k. K_v denotes the set of products that ship v can carry.

Loading As a ship v travels from (i,m) to (j,n), the quantity of product k onboard at the moment of departure from (j,n) is equal to the quantity loaded at (j,n) plus the quantity on board when it left (i,m) to (j,n): l_{imvk}. This constraint should hold true whenever ship v travels from (i,m) to (j,n); i.e., $x_{imjnv} = 1$.

$$x_{imjnv}(l_{imvk} + J_{jk}q_{jnvk} - l_{jnvk}) = 0, \text{ for every } v \in V, (i,m,j,n,k) \in A_v \times K_v. \quad (6)$$

Constraints (6) are nonlinear and are linearised in (Christiansen, 1999; Al-Khayyal & Hwang, 2007) as follows:

$$l_{imvk} + J_{jk}q_{jnvk} - l_{jnvk} + CAP_{vk}x_{imjnv} \le CAP_{vk}, \text{ for every } v \in V, (i,m,j,n,k) \in A_v \times K_v. \quad (7)$$

$$l_{imvk} + J_{jk}q_{jnvk} - l_{jnvk} + CAP_{vk}x_{imjnv} \ge -CAP_{vk}, \text{ for every } v \in V, (i,m,j,n,k) \in A_v \times K_v. (8)$$

Initial ship loading The following constraints ensure that $l_{i_v m_v vk}$, the quantity of product k on board v at the moment of departure from its initial position (i_v,m_v), is the sum of the initial quantity v already on board, Q_{vk} plus (if $J_{i_v v}$ = +1 and minus if J = -1) the quantity loaded (unloaded if J = -1) at the initial position $q_{i_v m_v vk}$.

$$Q_{vk} + J_{i_v k}q_{i_v m_v vk} - l_{i_v m_v vk} = 0, \text{ for every } v \in V, k \in K_v. \quad (9)$$

Non-ship-arrival inventory Constraints (10) require that if ship v does not arrive at (i,m), then no product can be loaded/unloaded onto/from v at (i,m); *i*.e., $q_{imvk} = 0$. Similar constraints, (11) are formulated for the initial conditions. Constraints (10) and (11) serve to tighten the formulation such that the departure time will equal the arrival time in (15) whenever there is no arrival at (i,m). These constraints allow simplification of the model by allowing simpler inventory level constraints at the end of the planning horizon (see constraints (22)).

$$CAP_{vk} \sum_{jn \in S_T} x_{jnimv} \ge q_{imvk}, \text{ for every } k \in K_v, (i,m) \in S_N, v \in V. \quad (10)$$

Compartment capacity Constraints (12) ensure that l_{imvk}, the quantity of product k on board v at the moment of departure from (i,m) does not exceed the ship's physical compartment capacity, CAP_{vk} for that product. l_{imvk} is set to 0 if there are no ships visiting (i, m);

i.e., $\sum_{(j,n)\in S_T} = 0.$ (11)

$$l_{imvk} \leq \sum_{(j,n)\in S_T} CAP_{vk} x_{jnimv}, \text{ for every } v \in V, (k,i,m) \in K_V \times S_N. \quad (12)$$

Product servicing The product servicing variable, o_{imvk} indicates whether product k is loaded or unloaded at (i,m). o_{imvk} needs to take on a value of 1 if k is loaded/unloaded at (i,m), and 0 otherwise. This is expressed as the following constraints.

$$q_{imvk} \leq CAP_{vk} o_{imvk} \text{ for every } v \in V, (k,i,m) \in K_V \times S_T. \quad (13)$$

3.3 Timing

A number of parameters are introduced to define the arrival and departure times. t_{im} denotes the time of arrival at position (i,m) while t_{Eim} denotes the departure time. TQ_{ik} represents the time required to load/unload one unit of product k at harbour i. W_i is the set-up time to service any product at harbour i. T_{ijv} is the time it takes for ship v to travel from harbour i to j.

Service time sequence Constraints (14) require that the m^{th} visit take place after the $(m\text{-}1)^{th}$ departure. This is a deviation from (Al-Khayyal & Hwang, 2007) which requires the m^{th} arrival take place after the $(m\text{-}1)^{th}$ arrival. This modification is to allow only one ship at a harbour at any one time.

$$t_{im} - t_{Ei(m-1)} \geq 0, \text{ for every } (i,m) \in S_N. \quad (14)$$

Service time Constraints (15) relate the arrival and departure times at position (i,m). The departure time, t_{Eim} from position (i,m) is equal to the arrival time, t_{im}, plus the service and setup times for all products.

$$t_{im} + \sum_{v\in V}\sum_{k\in K_v} TQ_{ik} q_{imvk} + W_i \sum_{v\in V}\sum_{k\in K_v} o_{imvk} - t_{Eim} = 0, \text{ for every } (i,m) \in S_T. \quad (15)$$

Routing and scheduling Constraints (16) relate the departure time from position (i,m) to the arrival time at (j,n) whenever ship v travels between those harbours; i.e., $x_{imjnv} = 1$.

$$x_{imjnv}(t_{Eim} + T_{ijv} - t_{jn}) \leq 0, \text{ for every } v \in V, (i,m,j,n) \in A_v. \quad (16)$$

Constraints (16) are nonlinear and are linearised in (Al-Khayyal & Hwang, 2007) as follows:

$$t_{Eim} + T_{ijv} - t_{jn} + 2Tx_{imjnv} \leq 2T, \text{ for every } v \in V, (i,m,j,n) \in A_v. \quad (17)$$

3.4 Inventory levels

Inventory constraints describe the relationship between quantities at harbours before and after each arrival to the quantities on board ships when they arrive to and depart from harbours. s_{imk} denotes the quantity of product k in harbour i at the time of the m^{th} arrival while s_{Eimk} is the quantity at the time of departure. K_i^H is the set of products serviced by harbour i. If J_{ik} = -1, R_{ik} is the rate at which product k is consumed at harbour i. If J_{ik} = +1, then R_{ik} is the rate of production. S_{MNik} is the minimum allowable stock level of product k at harbour i, while S_{MXik} is the maximum.

Initial inventory levels Constraints (18) require that the quantity of product k at the time of the first arrival to harbour i be equal to the initial inventory, IS_{ik} plus (if J_{ik} = +1 or minus if J_{ik} = -1) the amount produced (or consumed) up until the first arrival.

$$s_{i1k} = IS_{ik} + J_{ik} R_{ik} t_{i1}, \text{ for every } (i,k) \in H_N \times K_i^H. \quad (18)$$

For harbours that already have ships at the beginning of the planning horizon, H_0, $t_{i1} = 0$ such that $s_{i1k} = IS_{ik}$.

Inventory levels between arrivals and departures The following constraints enforce the inventory levels between the time of arrival of ship v to harbour i at position m and the time of departure from that position. The inventory level at departure is equal to the inventory level at arrival plus (if J_{ik} = +1, or minus if J_{ik} = -1) the amount of product k produced (or consumed if J_{ik} = -1) between the arrival and departure times.

$$s_{imk} = \sum_{v \in V} J_{ik} q_{imvk} + J_{ik} R_{ik} (t_{Eim} - t_{im}) - s_{Eimk} = 0, \text{ for every } (i,m,k) \in S_T \times K_i^H. (19)$$

Inventory levels between arrivals Constraints (20) enforce the inventory levels between two successive arrivals to harbour i. The inventory level of product k at the time of the m^{th} arrival to harbour i, s_{imk} is equal to the inventory level at departure from the previous position (m - 1), $s_{Ei(m-1)k}$ plus (if J_{ik} = +1, or minus if J_{ik} = -1) the amount produced (if J_{ik} = +1) or consumed (if J_{ik} = -1) between those times. A previous formulation used a non-linear constraint to allow multiple ships to be in one harbour at the same time. This however, makes it difficult to add constraints later to track and limit the number of ships in one harbour. As this research will require modeling of harbours with multiple berths, another formulation is adopted, which is presented but not used in (Al-Khayyal & Hwang, 2007). Therefore, any linear reformulation and their associated additional variables are no longer necessary.

$$s_{Ei(m-1)k} + J_{ik} R_{ik} (t_{im} - t_{Ei(m-1)}) - s_{imk} = 0, \text{ for every } (i,m,k) \in S_N \times K_i^H. \quad (20)$$

Inventory level bounds Constraints (21) was presented in (Al-Khayyal & Hwang, 2007) to ensure that inventory levels at harbours are within bounds. The term J_{ik} R_{ik} $(T - t_{Eim})(y_{i(m+1)} - y_{im}) \leq S_{MXik}$ is activated whenever y_{im} is the last arrival; i.e., $y_{i(m+1)} - y_{im} = 1$. This however, requires that additional constraints be formulated such that there exists $y_{i(m+1)} - y_{im} = 1$ for every harbour.

$$s_{Ei(m-1)k} + J_{ik} R_{ik} (t_{im} - t_{Ei(m-1)}) - s_{imk} = 0, \text{ for every } (i,m,k) \in S_N \times K_i^H. \quad (21)$$

Constraints (23) to (24), adopted from (Christiansen, 1999) are used instead to ensure that inventory levels remain within the required limits at harbour i at all times within the planning horizon. Constraints (10), (11) resulted in a tighter formulation which ensure that l_{imvk}, $q_{imvk} = 0$ if there are no arrivals to (i,m), and as such t_{imk}, t_{Eimk}, s_{imk}, s_{Eimk}, are correct from (15), (24). By eliminating (21), a non-linear constraint, the associated linear reformulation and additional variables are no longer necessary.

$$S_{MNik} \le s_{imk} \le S_{MXik}, \text{ for every } (i,m,k) \in S_T \times K_i^H. \tag{22}$$

$$S_{MNik} \le s_{Eimk} \le S_{MXik}, \text{ for every } (i,m,k) \in S_T \times K_i^H. \tag{23}$$

$$S_{MNik} \le s_{Eimk} + J_{ik} R_{ik}(T - t_{Eim}) \le S_{MXik}, \text{ for every } i \in I, k \in \times K_i^H, m = \mu_i. \tag{24}$$

3.5 Objective function

As in (Christiansen, 1999; Al-Khayyal & Hwang, 2007) the objective function is set to minimise the total operating cost over the planning horizon. The total operating cost is comprised of traveling costs and loading/unloading cost. Traveling costs include fuel and ship operating costs, while loading/unloading costs include port operations, duties, agent fees, berthing charges, etc. C_{ijv} denotes the total traveling cost for ship v from harbour i to j and is assumed to be independent of all other variables. C_{Wik}, on the other hand is the loading/unloading costs of product k at harbour i and is again assumed to be independent of all other variables. The objective function therefore, is to minimise (25) subject to constraints presented earlier.

$$\sum_{v \in V} \sum_{(i,m,j,n) \in A_v} C_{ijv} x_{imjnv} + \sum_{(i,m) \in S_T} \sum_{v \in V} \sum_{k \in K_v} C_{Wik} o_{imvk} \tag{25}$$

3.6 Summary of extensions to formulations in literature

This following summarizes modifications to formulations in literature.

1. In order to prevent ships from being routed to another arrival number within the same harbour, constraints (4) were added to the formulation.

2. Constraints (10) and (11) were added to ensure that $q_{imv} = 0$ if ship v does not make any visits to position (i,m) throughout the planning horizon. A positive value for q_{imv} even though there is no ship arrival to position (i,m) by ship v leads to an incorrect value for l_{jnvk} due to constraints (7) and (8), as well as incorrect values for t_{Eim} due to constraints (15). An incorrect value for t_{Eim} in turn, would lead to incorrect values for $s_{Ei(m-1)}$ due to constraints (20). Incorrect values for the departure time, inventory levels at harbour and onboard thereby, leads to an invalid schedule.

3. These constraints (21) rely on the expression $y_{i(m+1)} - y_{im}$ being equal to 1 at some position (i,m) in each harbour. Due to constraints (3), y_{im} has been constrained to be equal to 0 whenever there is a ship arrival and 0 otherwise. Therefore, the expression $y_{i(m+1)} - y_{im}$ is expected to equal to 1 whenever (i,m) is the last position visited. Constraints (3), however only applies for positions $(i,m) \in S_N$, leaving the initial positions y_{im}, $(i,m) \in S_0$ free to take on either 1 or 0. This may lead to the expression $y_{i(m+1)} - y_{im}$ not being equal to 1 if there are no ship arrivals to that harbour. Therefore, constraints (21) will not necessarily be activated to check inventory levels at the end of the planning horizon for this harbour. Rectifying this would require additional constraints to ensure that $y_{i(m+1)} - y_{im} = 1$ exists for each harbour. However,

another modification is proposed as follows which serves to alleviate the problems above and result in a simpler overall formulation.

Constraints (21) are removed (along with ten linearised equations and two associated variables, and replaced with constraints (23) and (24). Using constraints (24) removes the requirement of introducing additional constraints to ensure that $y_{i(m+1)} - y_{im} = 1$ exists for each harbour. As constraints (10) and (11) have been introduced, departure times t_{Eim} would equal the previous departure time if there are no arrivals at (i,m). This means that t_{Eim} at the last position is equal to the last departure time from that harbour. Constraints (24) therefore check the inventory level at the end of the planning horizon.

4. In order to allow only one ship to dock at any harbour at any one time, the time sequence constraints, (14) were modified accordingly such that a ship's arrival takes place only after the previous ship has departed. These modifications are expected to allow easier modification of the formulation to allow a more realistic representation multiple berths in the future. Constraints (20) are linear and do not necessitate additional decision variables.

4. Example Problem and Results

A small example problem as described in (Al-Khayyal & Hwang, 2007) is adopted for the purpose of testing and validating the formulation in Section 3. The example problem is summarised in the following paragraph.

The distribution system consists of 2 ships ($V = \{1, 2\}$) traveling between 3 harbours ($H_T = \{1, 2, 3\}$). Each harbour services 2 products ($K_i^H = \{1, 2\}, i \forall H_T$). Parameters for harbours and ships are summarised in Tables 1 and 2 respectively. Traveling time is assumed to take 0.3 days between any two harbours $T_{ijv} = 0.3,\ \forall i, j \in H_T, i \neq j, v \in V$. For ship 1, it costs \$1 to travel between any two harbours ($C_{ij1} = \$1,\ \forall i, j \in H_T, i \neq j$), while it costs \$1.50 for ship 2 ($C_{ij2} = \$1.50, \forall i, j \in H_T, i \neq j$). Further, it costs \$0.50 to load or unload one unit of any product at any harbour ($C_{Wik} = \$0.50,\ \forall i \in H_T, k \in K_i^H$). Unit loading/unloading time for any product at any harbour is 0.01 days ($TQ_{ik} = 0.01,\ \forall i \in H_T, k \in K_i^H$) while set up time is 0 ($W_i = 0,\ \forall i \in H_T$).

Table 1 Harbour Parameters

Harbour, i	1		2		3	
Product, k	1	2	1	2	1	2
Inventory lower bound, S_{MNik}	0	0	0	0	0	0
Inventory upper bound, S_{MXik}	20	30	20	20	20	20
Production/consumption rate, $J_{ik}R_{ik}$	-10	20	5	-10	5	-15
Initial inventory, IS_{ik}	10	15	5	15	10	15

Table 2 Ship Parameters

Harbour, i	1		2	
Product, k	1	2	1	2
Inventory lower bound, S_{MNik}	0	0	0	0
Inventory upper bound, S_{MXik}	20	30	20	20
Production/consumption rate, $J_{ik}R_{ik}$	-10	20	5	-10

The problem is solved to optimality using the linear optimisation suite ILOG CPLEX on a personal computer and a solution time of less than a second was obtained. Results for decision variables are presented in Table 3.

Table 3 Results for Decision Variables

x_imjnv		jn (1,1)		(1,2)		(1,3)		(2,1)		(2,2)		(2,3)		(3,1)		(3,2)		(3,3)		y
im	(1,1)	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	(1,2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	(1,3)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	(2,1)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	(2,2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	(2,3)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	(3,1)	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	(3,2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	(3,3)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

	z_imv		o_imvk v=1		o_imvk v=2		t	s_imk		q_imvk v=1		q_imvk v=2		t_E	sE_imk		l_imvk v=1		l_imvk v=2	
im	v=1	v=2	k=1	k=2	k=1	k=2		k=1	k=2	k=1	k=2	k=1	k=2		k=1	k=2	k=1	k=2	k=1	k=2
(1,1)	0	0	0.00	1.00	0.00	0.00	0.00	10.00	15.00	0.00	5.00	0.00	0.00	0.05	9.50	11.00	0.00	5.00	0.00	0.00
(1,2)	0	0	0.00	0.00	1.00	1.00	0.40	6.00	18.00	0.00	0.00	10.00	20.00	0.70	13.00	4.00	0.00	0.00	0.00	20.00
(1,3)	0	0	0.00	0.00	0.00	0.00	0.70	13.00	4.00	0.00	0.00	0.00	0.00	0.70	13.00	4.00	0.00	0.00	0.00	0.00
(2,1)	1	0	0.00	1.00	0.00	0.00	0.35	6.75	11.50	0.00	5.00	0.00	0.00	0.40	7.00	16.00	0.00	0.00	0.00	0.00
(2,2)	0	0	0.00	0.00	0.00	0.00	0.40	7.00	16.00	0.00	0.00	0.00	0.00	0.40	7.00	16.00	0.00	0.00	0.00	0.00
(2,3)	0	0	0.00	0.00	0.00	0.00	0.40	7.00	16.00	0.00	0.00	0.00	0.00	0.40	7.00	16.00	0.00	0.00	0.00	0.00
(3,1)	0	0	0.00	0.00	1.00	0.00	0.00	10.00	15.00	0.00	0.00	10.00	0.00	0.10	0.50	13.50	0.00	0.00	10.00	0.00
(3,2)	0	1	0.00	0.00	0.00	1.00	1.00	5.00	0.00	0.00	0.00	0.00	20.00	1.20	6.00	17.00	0.00	0.00	0.00	0.00
(3,3)	0	0	0.00	0.00	0.00	0.00	1.20	6.00	17.00	0.00	0.00	0.00	0.00	1.20	6.00	17.00	0.00	0.00	0.00	0.00

Upon inspection, decision variables comply with all constraints as specified in Section 3 and an optimal transportation cost is obtained equivalent to literature. Further inspection reveals that ship movements and loading/unloading activities demonstrate sensible behaviour in the physical world. Furthermore, inventory levels for all products at all harbours are within capacity limits as shown in Figures 2 to 7. An example interpretation of the decision variables at t = 0.10 day is described in the following paragraph.

At time, t = 0.10 day (Figure 1), Ship 1 is en route to (i,m) = (2,1), while Ship 2 has just completed loading and is routed to (i,m) = (1,2) from Harbour 1. Inventory levels during departure of Ship 2 at Harbour 3 are still within bounds (s_{311}= 0.50, s_{312} = 13.50). The complete interpretation of the decision variables are found in **Appendix A**.

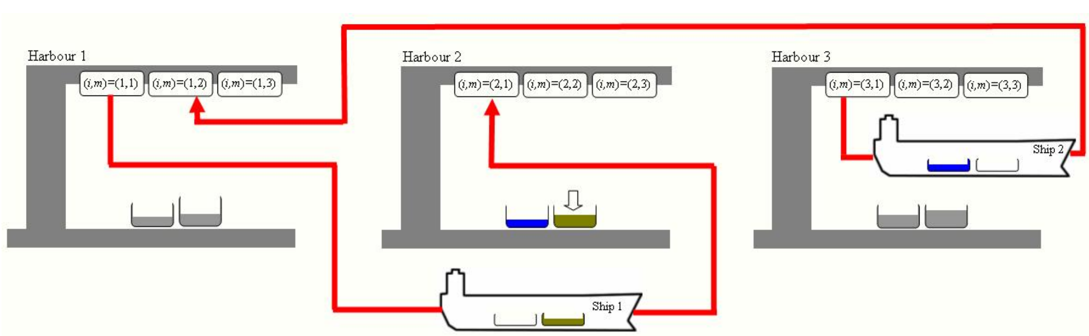

Figure 1 Time, t = 0.10 Day

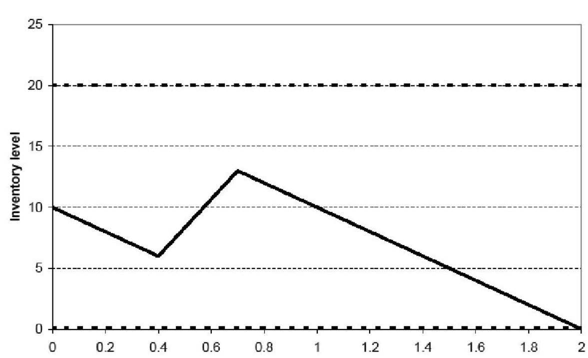

Figure 2 Product 1 in Harbour 1

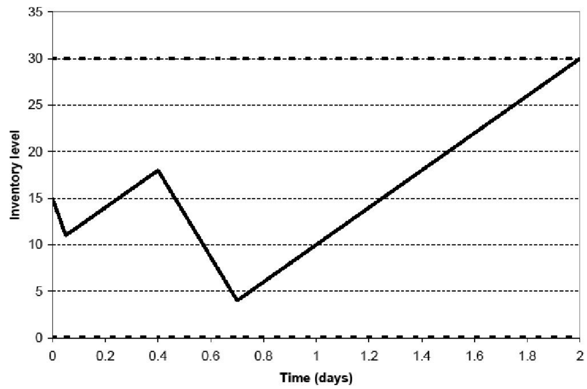

Figure 3 Product 2 in Harbour 1

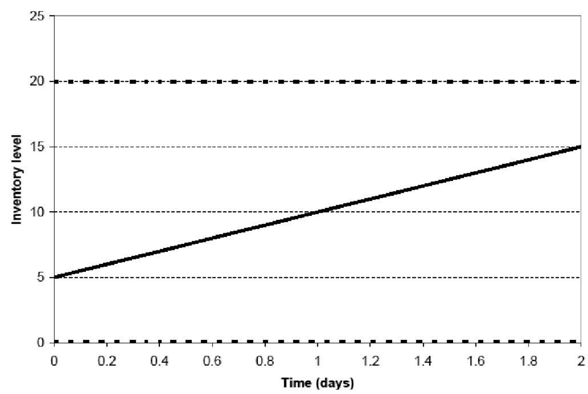

Figure 4 Product 1 in Harbour 2

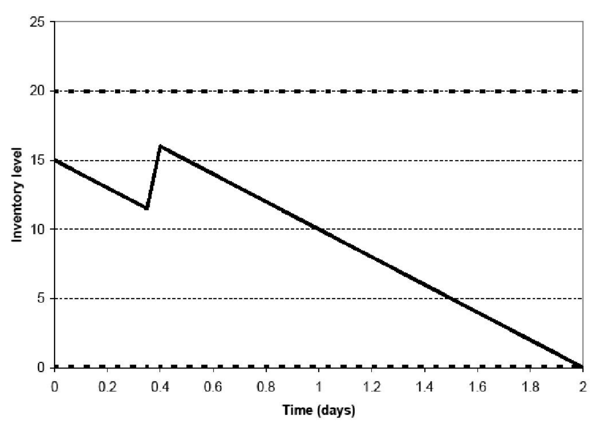

Figure 5 Product 2 in Harbour 2

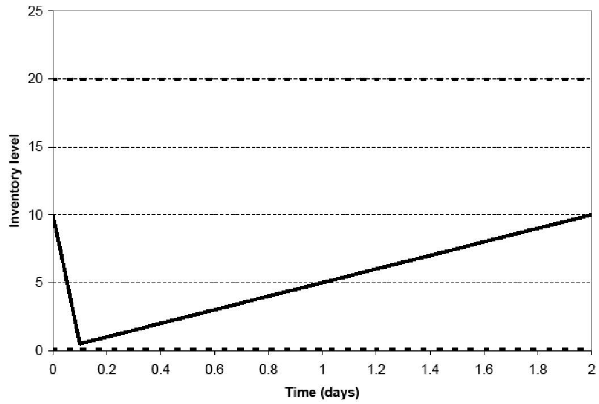

Figure 6 Product 1 in Harbour 3

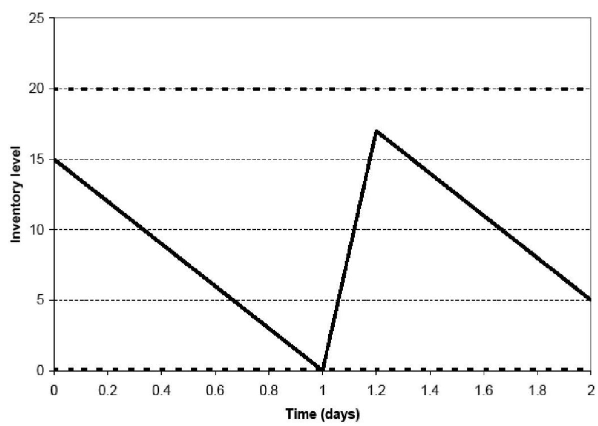

Figure 7 Product 2 in Harbour 3

6. Computation Load Results

To understand the model's ability to handle realistic instances, a number of numerical experiments were performed on test instances. Instances were generated by gradually increasing the case complexity through increased available positions, number of products, number of harbours and number of ships in the distribution system.

First, the test case in Section 4 is taken as the base case and the number of available positions μ is added successively while randomly modifying other instance parameters. 20 random instances were tested over μ values of 3, 4, 5 and 6. Average solving times are shown in Figure 8.
Second, another 20 random instances were generated over increasing number of products, K. Figure 9 show average solving times for varying K.

Finally, another 20 instances were generated for increasing number of ships, V. As the number of ships is increased, the number of harbours is to be serviced, H. In the convention of the original test example above, the number of ships kept to one less than the number of harbours. Figure 10 indicates the solving times with respect to increasing number of ships and harbours.

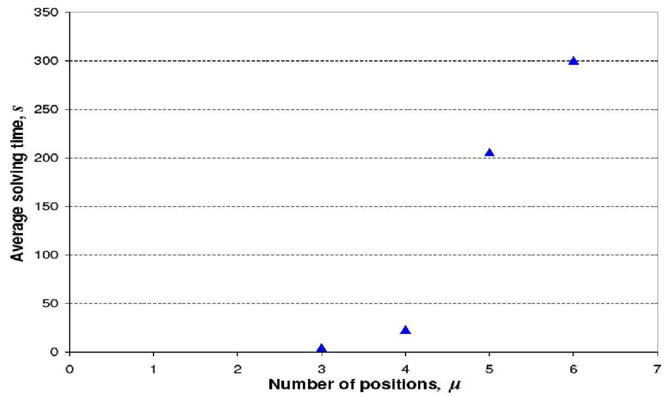

Figure 8 Solving Times as μ is Increased

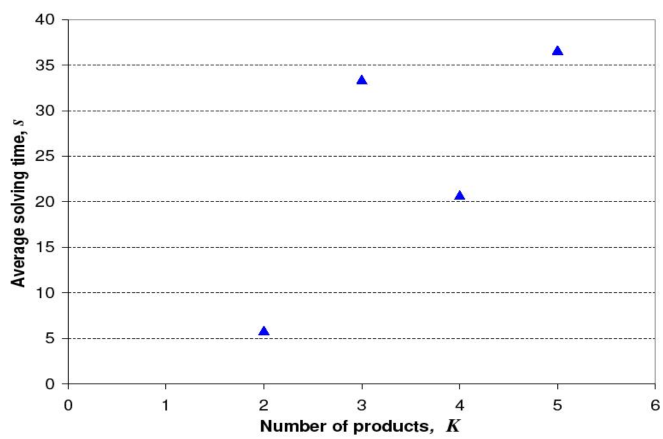

Figure 9 Solving Times as K is Increased

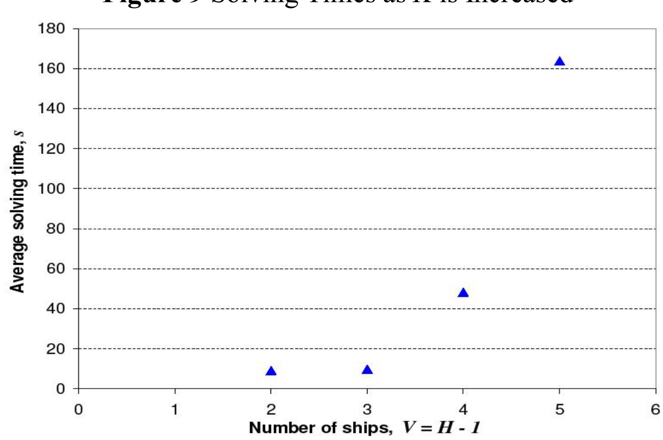

Figure 10 Solving Times as $V = H - 1$ is Increased

Figures 8, 9 and 10 indicate substantial increases in the average solving times with increasing complexity of the case instance, although the effect is less pronounced when the number of products is increased. Al-Khayyal and Hwang (Al-Khayyal & Hwang, 2007) reported an exponential increase in solving time as the number of available positions is increased. However, it is noted that there is great variability in the solving times even for cases with the same level of complexity (same number of μ, K , V or H).

7. Conclusion

In this research, a mathematical formulation was developed and then solved based on a problem found in literature. An optimal solution was obtained and ship movements and loading/unloading activities demonstrated sensible behaviour. This formulation contains a number of extensions to improve the current formulation, simplify the current formulation in literature by removing unnecessary non-linear constraints and allow the model to be extended for more realistic multiple berths.

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