Heuristic Methods for Capacitated Vehicle Routing Problem

by

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Abstract

The Capacitated Vehicle Routing Problem (CVRP) is a well known problem in the optimization literature. In this paper, we consider that there is a single depot (or distribution centre) that caters to the customer demands at a set of sales points (or demand centers) using vehicles with known limited capacities. The demand at each of these demand centers is assumed to be constant and known. Due to its limited capacity, the vehicles may need to make several trips from the depot for replenishment. The problem is to determine an optimal distribution plan that meets all the demands at minimum total cost. We propose a heuristic based on the well-known column generation approach to solve this problem. A sub-problem is then solved to generate additional columns for the master problem. This paper examines an approach for generating "good" columns (or distribution plans) together with heuristics for sequencing the customers within each distribution plan. The results are then compared to those obtained by using the genetic algorithm to demonstrate the performance of the methods.

Keywords: Column Generation, Complete Enumeration, Genetic Algorithm, Capacitated Travelling Salesman

1. Introduction

The Capacitated Vehicle Routing Problem (CVRP) requires the determination of an optimal set of routes for a set of vehicles to serve a set of customers. The problem as it appears in real life may have several additional constraints, such as limits on the capacity of the vehicles, time windows for the customer to be served, limits on the time that a driver can work, limits on the lengths of the routes, etc. The problem was first introduced by Dantzig and Ramser, 1959. Due to the intrinsic interest as a difficult combinatorial optimization problem and to the economic importance of applications, CVRP has received a lot of attention and many algorithms, both exact and heuristic, have been developed since then to solve the general problem as well as real world cases, and so literature on the subject is very extensive. An excellent survey of these techniques can be found in Laporte and Semet, 1999.

As in most NP-hard problems, three approaches are typically employed to solve these types of problems: heuristics, approximation methods and exact methods. While heuristics do not provide guarantees about the solution quality, they are useful in practical contexts because of their speed and ability to handle large instances. A special class of heuristics is meta-heuristics, which are general frameworks for heuristics. Approximation algorithms are a special class of heuristic that provides a solution and an error guarantee. Exact methods guarantee that the optimal solution is found if the method is given sufficiently time and space.

This paper deals with the following variant of the Capacitated Vehicle Routing Problem (CVRP): a single depot where routes begin and end provides service to a set of customers and their known demands, with a single vehicle or several similar vehicles with a maximum weight that each can load, and distances (or costs) between customers and between customers and the depot. It is assumed that distances satisfy triangular inequality. We want to find routes for the vehicles starting and ending at the depot that satisfy the demand at a minimum total cost.

2. Literature Review

Dantzig and Ramser, 1959 consider the routing of a fleet of gasoline delivery trucks between a bulk terminal and a number of service stations supplied by the terminal. The distance between any two locations is given and a demand for a certain product is specified for the service stations. The problem is to assign service stations to trucks such that all station demands are satisfied and total mileage covered by the fleet is minimized. The authors imposed the additional conditions that each service station is visited by exactly one truck and that the total demand of the stations supplied by a certain truck does not exceed the capacity of the truck. The problem formulated was given the name 'truck dispatching problem'. The name VRP is more recent and the variant of the VRP is given the name 'Capacitated Vehicle Routing Problem (CVRP)'.

Several families of heuristics have been proposed for the Vehicle Routing Problem. These can be broadly classified as: classical heuristics developed mostly between 1960 and 1990, and metaheuristics whose growth has occurred over the last two decades. Most standard construction and improvement procedures that are in use today can be termed classical. These methods perform a relatively limited exploration of the search space and typically produce good quality solutions within modest computing times.

Classical VRP heuristics can be further classified into three broad categories: Construction heuristics, Two-phase heuristics and Improvement heuristics. Construction Heuristics build a feasible solution while trying to minimize solution cost, but do not contain an improvement phase. Two phase heuristic decomposes the problem is into its two natural components: clustering of vertices (customers) into feasible routes and actual route construction. Two-phase heuristics can be of two types: cluster first, route second methods and route first, cluster second method. In the first case, vertices are first organized into feasible clusters, and a vehicle route is constructed for each of them. In the second case, a tour is first built on all vertices and it is subsequently segmented into feasible vehicle routes. Finally, improvement heuristics attempt to upgrade any feasible solution by performing a sequence of edge or vertex exchanges within or between vehicle routes. An excellent discussion of classical heuristics for the VRP can be found in Bodin et al., 1983, Golden and Assad, 1988 and Fisher, 1995.

3. Problem Formulation

The following VRP formulation is due to Fisher and Jaikumar, 1981.

Constants
K= number of vehicles
n= no. of customer to which a delivery must be made. Customers are indexed from 1 to n and index 0 denotes the central depot.
b_k = capacity of vehicle K.
a_i = demand of customer i
c_{ij} = cost of direct travel from customer i to customer j.

Variables

$$y_{ik} = \begin{cases} 1, & \textit{if the order from customer } i \textit{ is delivered by vehicle } k. \\ 0, & \textit{otherwise} \end{cases}$$

$$x_{ijk} = \begin{cases} 1, & \textit{if the vehicle } k \textit{ travels directly from customer } i \textit{ to customer } j \\ 0, & \textit{otherwise} \end{cases}$$

An integer programming formulation of the problem of routing to minimize cost subject to vehicle capacity constraint is given below.

Minimize

$$\sum_{ijk} c_{ij} x_{ijk} \qquad (1)$$

subject to

$$\sum_{i} a_i y_{ik} \le b_k \qquad k = 1, \ldots, K \qquad (2)$$

$$\sum_{k} y_{ik} = \begin{cases} K, & i = 0 \\ 1, & i = 1, \ldots, n \end{cases} \qquad (3)$$

$$y_{ik} = 0 \text{ or } 1, \qquad \begin{cases} i = 0, \ldots, n \\ k = 1, \ldots, K \end{cases} \qquad (4)$$

$$\left.\begin{array}{lll}
\sum_{i} x_{ijk} = y_{jk}, & j = 0,\dots,n & (5) \\
\sum_{j} x_{ijk} = y_{ik}, & i = 0,\dots,n & (6) \\
\sum_{i,j\in S\times S} x_{ijk} \le |S| - 1, & \begin{cases} S \subseteq \{1,\dots,n\} \\ 2 \le |S| \le n-1 \end{cases} & (7) \\
x_{ijk} = 0 \text{ or } 1, & \begin{cases} i = 0,\dots,n \\ j = 0,\dots,n \end{cases} & (8)
\end{array}\right\} \quad k = 1,\dots,K$$

Two well known combinatorial optimization problems are embedded within this formulation. Constraints (2)-(4) are the constraint of a generalized assignment problem and ensure that each route begins and ends at the depot (customer 0), that every customer is serviced by some vehicle, and that the load assigned to a vehicle is within its capacity. If the yik are fixed to satisfy (2) – (4), then for given k, constraint (5)-(8) define a travelling salesman problem over the customers assigned to vehicle k.

4. Computational Work

For the purpose of comparison of heuristics, we have considered two datasets for the VRP, in which the number of customers is 9 and 15 respectively, with the demands given for each customer. The distances between the customers, as well as from the depot to each customer, is also given. The objective is to minimize the overall transportation cost by satisfying the demand of each customer. Without loss of generality, we have considered the transportation cost proportional to the distance travelled. The two VRP instances were solved using three classical heuristics, namely (a) the parallel savings algorithm, (b) the generalized assignment heuristic, and (c) the location based heuristic.

Parallel Saving Algorithm (Altinkemer and Gavish, 1991) The algorithm is based on the idea of maximum savings that can be obtained by merging two trips. The original savings approach (Clarke and Wright, 1964) was used in the TSP context. In the VRP context, the sequential, single-tour merging procedure is replaced by a matching-based procedure that merges multiple partial solutions in each step.

Generalized Assignment Heuristic (Fisher and Jaikumar, 1981) An assignment of customers to vehicles is obtained by solving a generalized assignment problem with an objective function that approximates delivery cost. The algorithm is guaranteed to find a feasible solution, if one exists.

Location Based Heuristic (Bramel and Simchi-Levi, 1995) This heuristic is based on formulating the routing problem as a location problem commonly called the capacitated concentrator location problem (CCLP). This location problem is subsequently solved and the solution is mapped back to the original vehicle routing problem.

The same two problems were also solved by our proposed column generation approach based on Lagrangian multipliers obtained from a restricted master problem. Finally, for benchmarking purpose, the problems were solved using an enumeration approach that provides the optimal solution for these two instances. The results obtained by the five different methods are then compared. Finally solutions to the problems were also obtained by using the Genetic Algorithm approach developed for a multi-vehicle capacitated TSP (Vachajitpan, 2008).

5. Column Generation Based Heuristic

We propose a heuristic based on the well known column generation approach to solve the CVRP. In this approach, each column represents a distribution plan that services a subset of demand centers. When the vehicle capacity is large enough to satisfy all customer demands in a single tour, the problem reduces to the classical Travelling Salesman Problem (TSP). Due to the limited vehicle capacity, a decision needs to be made regarding the optimal assignment of demand centers to the distribution plans. Further, each distribution plan represents an embedded TSP that needs to be solved to determine the sequence of visits made to the customers by the vehicle.

Since the number of columns can be arbitrarily large, a restricted master problem is used to initiate the computations. A sub-problem is then solved to generate additional columns for the master problem. The sub-problem uses the dual price information from the master problem to generate promising (that is, those that can lead to savings) columns that are also feasible (that is, satisfy the vehicle capacity constraint). However, due to the inherent complexity of this combinatorial problem, there is no guarantee that the selected columns will constitute the optimal (in terms of highest savings) subset of columns forming the basis in the master problem. Further, even if we know that this set of columns is optimal, one needs to solve a TSP for each of these columns to generate the optimal sequence of visits and the corresponding optimal total cost for the subset of centers covered by each column. Since optimality is not guaranteed, one needs to rely on heuristics to obtain a reasonably good sequence of customer visits within a column. Here we examine an approach for generating "good" columns (or distribution plans) together with heuristics for sequencing the customers within each distribution plan.

Formulation

A restricted master problem is used to determine the cost-optimal set of columns from out of a set of columns that covers each customer exactly once.

Notation

S	:	Set of all customers, S = { i }
Q_i	:	Demand of customer, i, for $i \in S$
D_j	:	j^{th} distribution plan (column vector)
d_{ij}	:	individual elements of D_j with $d_{ij} = 1$, if D_j services customer i, and 0 otherwise
DP	:	$\{ j \mid D_j \text{ exists} \}$
X_j	:	{ 0, 1} representing the selection or otherwise of plan D_j.
K	:	Capacity of vehicle
C_j	:	TSP cost of plan D_j
TC	:	Total Cost
λ_i	:	Lagrangian multiplier for customer i

The formulation for the restricted master problem is as follows:
Minimum

$$\sum_{j \in DP} C_j X_j \qquad (9)$$

subject to:

$$\sum_{j \in DP} d_{ij} X_j = 1 \quad \text{for all } i \in S \qquad (10)$$

$$\sum_{i \in S} d_{ij} Q_i \le K \quad \text{for all } j \in DP \qquad (11)$$

Initially, each customer demand is satisfied by a dedicated trip, thus, D is an identity matrix with each column D_j having a value 1 in row i corresponding to customer i. During implementation, only the first constraint is explicitly stated in the master problem, while the second constraint is always guaranteed to be satisfied by the subproblem that generates d_{ij} for the new column. The master problem generates the dual prices that are used as Lagrangian multipliers for the subproblem. The subproblem is another optimization problem that generates the most promising column for improvement of solution to the master problem.

Subproblem formulation

Maximize

$$\sum_{i \in S} d_{ij} \lambda_i \qquad (12)$$

subject to

$$\sum_{i \in S} d_{ij} Q_i \le K \quad \text{for all } j \in DP \qquad (13)$$

The column generated by the subproblem is added to the master problem, and solved again. The revised dual prices are used to solve another subproblem. The process is repeated iteratively until no more columns can be found that can improve the solution to the master problem.

6. Genetic Algorithm Based Solution

The solutions for the problems were also obtained by using the Genetic Algorithm (Holland, 1975 and Goldberg, 1989) which is available with the Excel Solver software. Starting with an arbitrarily feasible solution, the improved solutions are obtained with successive rounds of computation until no further improvement is found. Since GA is a meta heuristic procedure, it does not guarantee to find the optimum. However, by starting from different solutions we can obtain very good final solutions for the problems. The best result by GA approach for problem 1 (for 9 customers) is the same as the minimum result obtained by complete enumeration approach. For problem 2 (for 15 customers) the best results were obtained for 5 and 6 trips (vehicles) as shown in Table 4.

7. Optimum Method: Complete Enumeration

In order to compare the results obtained by all above heuristics, there is a need to first generate the optimum solution so that the exact deviation can be measured. However, for the CVRP it is not possible obtain the exact solution for large problems. Hence, two small sized problems have been selected to produce optimum solution: Problem 1 is for 9 customers and Problem 2 is for 15 customers.

A complete enumeration approach is used for obtaining optimal solution. The algorithm works in two phases: In the first phase, all feasible columns, that is, those columns that satisfy the vehicle capacity constraint are generated. While generating the next permutation systematically, an efficient algorithm is used that minimizes the time spent in swapping elements. The number of customers in a trip can vary from 1 to n, however, the number of customers is restricted based on the vehicle capacity so as to generate only a small set of routes that are feasible. After the route is generated, another permutation routine is used upon the customers within this route to obtain the optimal TSP length of the route. In the second phase, we examine systematically all combinations of routes that ensure feasibility of VRP, that is, each customer must be visited and exactly once. Combinations that do not satisfy this criterion are discarded. Out of these combinations of routes, the one with the smallest overall cost is identified as the optimal route. The entire logic was coded and tested in "C" Language.

8. Computational Results

Computational tests were performed using two test problems taken from the literature. Summary characteristics of the two problems are given in the following Tables.

Table 1 Summary of Problem Characteristic

Problem	No. of Customer	Demand of each customer	No. of Vehicle	Maximum Capacity of the vehicle
1	9	1 unit each	3	4 unit
2	15	Refer Table 2	5 and 6	200 units

Table 2 Demand of Each Customer for Problem 2

Customer No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Demand in Units	50	80	20	100	45	70	35	55	60	75	80	120	65	15	90

Comparison of heuristics for the test problems1 and 2 can be seen in Tables 3 and 4. Relative performance of each heuristics without considering the complexity of its computation can be seen from the limited test results.

Table 3 Comparison of Heuristics for Problem 1 (Customers 1 to 9)

Optimum Method by complete enumeration			Parallel Saving Heuristic			Generalized Assignment Heuristic (seed customer: 2,4,6)			Location Based Heuristic		
Route	Load	Distance	Route	Load	Distance	Route	Load	Distance	Route	Load	Distance
1,2,9	3	524	1,2,9	3	524	1,2,9	3	473	1,2,9	3	473
3,6	2	473	3,6	2	473	3,6	2	260	3,7,6	3	339
4,8,5,7	4	260	4,8,5,7	4	260	4,8,5,7	4	524	4,8,5	3	519
Minimum Distance		**1257**	Minimum Distance		**1257**	Minimum Distance		**1257**	Minimum Distance		1331

Column Generation based Heuristic			Genetic Algorithm Solution		
Route	Load	Distance	Route	Load	Distance
3	1	208	7,5,8,4	4	524
2,1,6,7	4	725	3,6	2	260
5,8,4,9	4	873	9,2,1	3	473
Minimum Distance		1806	Minimum Distance		**1257**

Table 4 Comparison of Heuristics for Problem 2 (Customers 1 to 15)

Optimum Method by complete enumeration(5 trips)			Optimum Method by complete enumeration(6 trips)			Parallel Saving Heuristic			Generalized Assignment Heuristic (seed customer: 2,4,10,11,12,15)		
Route	Load	Distance	Route	Load	Distance	Route	Load	Distance	Route	Load	Distance
8,2,9	195	1190	1,14,3,5, 9	190	1560	1,10,13	190	820	1,15	140	760
10,15	165	790	4,8	155	750	2,6	150	1100	2,6,5	195	1440
11,12	200	760	6,2,7	185	1110	3,14,5,1 5	170	1780	9,3,14, 10	170	1150
7,4,13	200	980	10	75	300	4,8	155	750	4,8,7	190	790
1,14,3,5,6	200	1660	13,15	155	690	7,9	95	610	11	80	480
			11,12	200	760	11,12	200	760	12,13	185	770
Minimum Distance (5 trips)		**5380**	Minimum Distance (6 trips)		**5170**	Minimum Distance		5820	Minimum Distance		5390

Column Generation based Heuristic			Genetic Algorithm Solution (5 Trips)			Genetic Algorithm Solution (6 Trips)		
Route	Load	Distance	Route	Load	Distance	Route	Load	Distance
1,14,3,5,6	200	1660	5,6,2	195	1440	12,11	200	760
8,2,9	195	1190	13,1,10	190	820	1,15	140	760
7,13,15	190	940	12,11	200	760	8,4,7	190	850
4,10	175	760	15,14,3,9	185	1600	10,3,14,9	170	1280
11,12	200	930	7,8,4	190	790	5,6,2	195	1440
						13	65	400
Minimum Distance (5 trips)		5480	Minimum Distance (5 trips)		5410	Minimum Distance (6 trips)		5490

9. Conclusion

The Capacitated Vehicle Routing Problem is a challenging unsolved problem and has attracted the attention of several researchers due to its immense practical importance. In this paper, we studied three classical heuristics and an optimal approach to this problem. In addition, we also propose a Column Generation Approach and the Genetic Algorithm for solving the CVRP. The performances of the heuristics are compared using 9 and 15-customer problems. The results indicate that no single heuristic is likely to give consistently better results, and there is scope for further research in this area that can improve upon these heuristics.

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